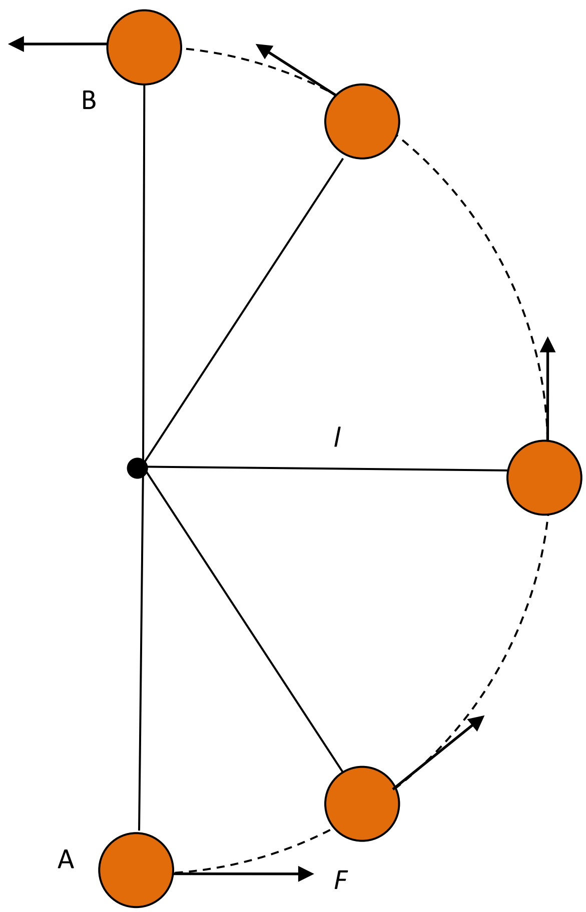
Teacher notes
Topic A

An instructive problem on circular motion and work done.

A ball of mass *m* hangs at the end of a vertical string of length *l* in position A. A force is applied to the ball so that the ball moves along a circular arc eventually reaching position B, where the string is again vertical. The force is always tangent to the vertical circular path.



The ball starts from rest at t = 0. The force is given by $F = mg(c + \cos\theta)$ where c is a constant. The angle is measured relative to a horizontal line, so it varies from $-\frac{\pi}{2}$ at A to $\frac{\pi}{2}$ at B.

(a) Determine the tension in the string at t = 0.
(b) Calculate the initial acceleration of the ball.
(c) Show that the tangential acceleration of the ball is constant.

(d) The string goes slack in position B. Show that $c = \frac{1}{2\pi}$.

(e) Determine the work done by force *F* from position A to position B.

AHL

(f) Determine

(i) the net torque on the ball.

(ii) the change in the angular momentum of the ball from A to B.

(iii) the time taken to move from A to B.

Answers

(a) $T - mg = m\frac{v^2}{l}$ but $v = 0$, so $T = mg$.

(b) At $t = 0$, $F = mg(c + \cos -\frac{\pi}{2}) = mgc$. Hence the acceleration is gc, directed to the right.

(c) $F - mg\cos\theta = ma_T$. Hence $a_T = \frac{F}{m} - g\cos\theta = g(c + \cos\theta) - g\cos\theta = gc$ and so constant.

(d) At the top, $T + mg = m\frac{v^2}{l}$, $T \to 0$ and so $v^2 = gl$. But also $v^2 = 2a_T l\pi = 2gcl\pi$ since $a_T = gc$ and the distance travelled from A to B is $l\pi$. Hence, $2gcl\pi = gl \Rightarrow c = \frac{1}{2\pi}$.

(e) From A to B: $W_{net} = \Delta E_K = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mgL$. $W_{net} = W_F + W_{mg} + W_T = W_F - mg(2l) + 0$.

Hence, $W_F - mg(2l) = \frac{1}{2}mgl$ and so $W_F = \frac{5}{2}mgl$.

(As a check: $W_F = \int_{-\pi/2}^{\pi/2} Fds = mg\int_{-\pi/2}^{\pi/2}(\frac{1}{2\pi} + \cos\theta)ld\theta = mgl(\frac{1}{2\pi} + \sin\theta)\Big|_{-\pi/2}^{\pi/2} = mgl(\frac{1}{2} + 2) = \frac{5mgl}{2}$.)

(f)

(i) Net torque is: $\tau = FL - mgl\cos\theta = mgl(c + \cos\theta) - mgl\cos\theta = mglc = \frac{mgl}{2\pi}$.

(ii) Change in angular momentum is $\Delta L = mvl - 0 = ml\sqrt{gl}$.

(iii) From $\tau = \frac{\Delta L}{\Delta t} \Rightarrow \Delta t = \frac{\Delta L}{\tau} = \frac{ml\sqrt{gl}}{mglc} = \frac{1}{c}\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{g}}$.

(This agrees with: $\Delta s = \frac{u+v}{2}\Delta t \Rightarrow \Delta t = \frac{2\Delta s}{0+v} = \frac{2\pi l}{\sqrt{gl}} = 2\pi\sqrt{\frac{l}{g}}$.)